

The mixing angle as a function of neutrino mass ratio

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In the quark sector, we experience a correlation between the mixing angles and the mass ratios. A partial realization of the similar tie-up in the neutrino sector helps to constrain the parametrization of masses and mixing, and hints for a predictive framework. We derive five hierarchy dependent textures of neutrino mass matrix with minimum number of parameters (≤ 4), following a model-independent strategy.

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The neutrino mass matrix plays the central role in the study of neutrino physics, as it contains the information of both the masses and mixing. In that sense, it is more fundamental than the PMNS matrix. It is always desired to derive a texture of the mass matrix which leads to significant prediction. If the neutrino mass matrix, \mathcal{M}_ν , follows μ - τ symmetry [1–9], we obtain two constraints on the matrix elements; they are: $(\mathcal{M}_\nu)_{12} = (\mathcal{M}_\nu)_{13}$ and $(\mathcal{M}_\nu)_{22} = (\mathcal{M}_\nu)_{33}$. These two constraints generate: $\theta_{13} = 0$ and $\theta_{23} = 45^\circ$. But the μ - τ symmetric texture does not tell anything about the neutrino mass hierarchy and solar angle. The texture becomes predictive only when it is associated with certain flavor symmetries [10–18]. On the contrary, the present experimental data strongly rule out any possibility of a vanishing reactor angle [19–21] and the central value of θ_{23} is more than 45° (and is close to 49°) [22]. These deviations undoubtedly questions the credibility of μ - τ symmetry.

Visualization of a more realistic neutrino mixing pattern and mass matrix, demands perturbation to the μ - τ symmetry [23–25]. In the present article we emphasize more on the possibility to perceive an exact texture of M_ν which is *model independent*, with minimum number of efficient elements, than following perturbation techniques. Our approach is bottom-up and inspired by the phenomenology of quark sector.

The Cabibbo angle (θ_c) [26, 27] is a parameter which plays a significant role in describing the quark masses and mixing. It is anticipated that this angle might be a function of the ratio of down and strange quark masses [28],

$$\sin \theta_c \simeq \sqrt{\frac{m_d}{m_s}}. \quad (1)$$

It is an esteemed endeavor of particle physicists to unify the quark and lepton sectors, or to realize similar kind of happenings in both the sectors otherwise. Based

on this, is it possible to extend a similar idea in the form of an *ansatz* in the neutrino sector also, as in the following,

$$\sqrt{\frac{m_i}{m_j}} = \sin \theta_{ij}. \quad (i, j = 1, 2, 3)? \quad (2)$$

Undoubtedly there are several hurdles which will arise both from the theoretical and phenomenological perspectives. The reason being the difference between the mixing mechanism in both the sectors. The CKM matrix is very close to unit matrix and the spectra of “up” and “down” quarks are strongly hierarchical. But, for neutrinos we are ignorant of the exact hierarchy of the masses. Unlike the quarks, the mixing is quite large in lepton sector and the PMNS matrix is far from being an unit matrix.

So long the reactor angle was predicted to be vanishing, such development in Eq.(2) seems obsolete. But, in the light of present data, when, $\theta_{13} \sim \theta_c$, one cannot deny the possible existence of the following relation,

$$\sqrt{\frac{m_1}{m_3}} \simeq \sin \theta_{13} = \epsilon, \quad (\text{say}), \quad (3)$$

in the non-degenerate spectrum of neutrino masses, obeying normal ordering (see Fig.(1)). We emphasize that the realization of the ansatz in Eq. (2) is not “full”, but “*partial*”. Because, it seems impossible to realize all the three possibilities (See in Eq. (2)) *simultaneously*. For example, if we realize Eq. (3), then another possibility,

$$\sqrt{\frac{m_2}{m_3}} \simeq \sin \theta_{23} = \eta; \quad (4)$$

is ruled out and vice-versa (See Fig.(1)). Again, a similar realization in 1-2 sector is forbidden because of the smallness of the solar mass squared difference, which insists the mass ratio, $m_1 : m_2$ to be constant, and this ratio tends towards unity.

Because of similar reasons, for inverted ordering of the neutrino masses, only the 2-3 realization,

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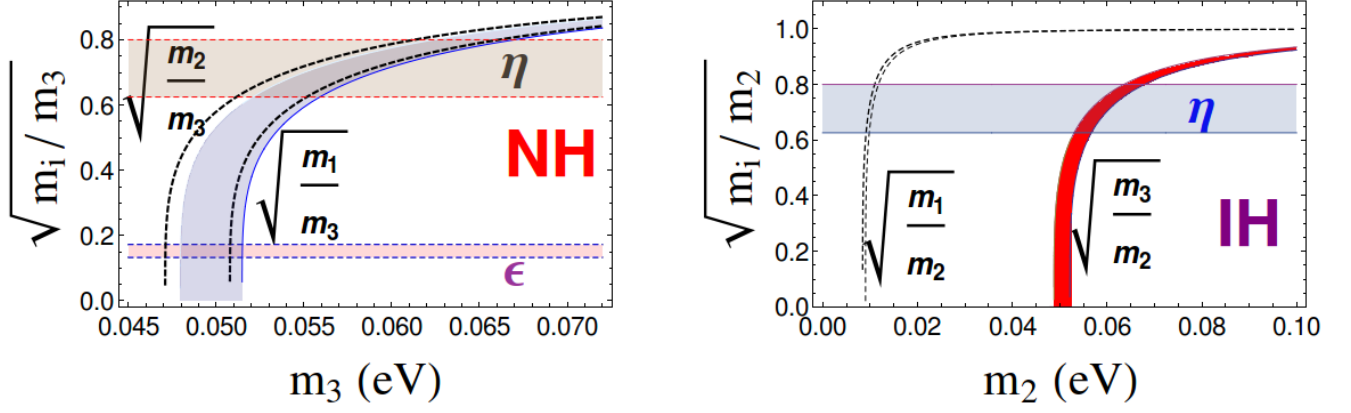


FIG. 1. The evolution of the neutrino mass ratios with respect to the absolute mass scale for both normal (left) and inverted ordering (right) of the neutrino masses (Corresponding to 3σ range of Δm_{21}^2 and Δm_{31}^2) are illustrated.

$$\sqrt{\frac{m_3}{m_2}} \simeq \sin \theta_{23} = \eta; \quad (5)$$

is possible.

Now even though there are several clampdowns, a partial realization of the ansatz (Eq. (2)) clutches certain positive aspects like it puts some constraints on the parametrization of neutrino masses and mixing. Let us discuss some implications of the ansatz in Eq. (2).

- It categorizes the parametrization in both ways: “ ϵ -based” or “ η -based”.
- The “ ϵ -based” parametrization only encompasses the Normal ordering of the masses with non-degenerate spectrum (**NH-ND**), with absolute mass scale, $0.047 \text{ eV} \leq m_3 \leq 0.05 \text{ eV}$. This parametrization rules out any possibility of vanishing m_1 (Strict-NH case). Since the reactor angle is not zero as depicted in Eq. (3).
- “ η -based” parametrization encompasses both the Normal ordering and inverted ordering of neutrino masses with degenerate spectrum (**NH-QD** and **IH-QD**) only, with absolute mass scale, $0.05 \text{ eV} \leq m_3(m_2) \leq 0.067 \text{ eV}$.

- The present ansatz (Eq. (2)), rules out the possibility of non-degenerate inverted spectrum of neutrino masses.
- In the degenerate limit, the ansatz sets the upper limit on the sum of the neutrino masses, $\Sigma m_i \leq 0.17 \text{ eV}$. This prediction is relevant in the light of present Cosmological observation.

We can see that although the ansatz, (Eq. (2)) cannot solve the hierarchy issue, yet it can put some constraints on the mass spectrum. This are subjected to the sensitivities of the future experiments. The ϵ and η based mass spectrum can be represented in the following way,

$$M_\nu^d(\epsilon, s) = \begin{bmatrix} s\epsilon^2 & 0 & 0 \\ 0 & s\epsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} m_3, \quad (\text{NH-ND}) \quad (6)$$

$$M_\nu^d(\eta, c) = \begin{bmatrix} c\eta^2 & 0 & 0 \\ 0 & \eta^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} m_3, \quad (\text{NH-QD}) \quad (7)$$

$$M_\nu^d(\eta, c) = \begin{bmatrix} c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \eta^2 \end{bmatrix} m_2, \quad (\text{IH-QD}) \quad (8)$$

where, s and c are $\mathcal{O}(1)$ coefficients: $\epsilon < s < \epsilon^{-1}$, $\eta < c < \eta^{-1}$. The positivity of Δm_{21}^2 enforces, $c < 1$.

We are working in a basis, where the charged lepton mass matrix is diagonal. We represent the PMNS matrix, $U(\epsilon)$ which is “ ϵ ” motivated as in the following,

$$U(\epsilon, d, f) \approx \begin{bmatrix} 1 - \frac{1}{2}f^2\epsilon^2 - \frac{\epsilon^2}{2} & f\epsilon & \epsilon \\ -f\epsilon - d\epsilon^2 & 1 - \frac{1}{2}f^2\epsilon^2 - \frac{d^2\epsilon^2}{2} & d\epsilon \\ -\epsilon + cd\epsilon^2 & -d\epsilon - f\epsilon^2 & 1 - \frac{1}{2}d^2\epsilon^2 - \frac{\epsilon^2}{2} \end{bmatrix}, \quad (\text{PMNS-I}) \quad (9)$$

Where, d and f are $\mathcal{O}(1)$ coefficients. We put forward an-

other possible form of PMNS matrix, which is motivated

by η -based parametrization, $U(\eta)$,

$$U(\eta, b, c) \approx \begin{bmatrix} \sqrt{\frac{2}{3}}c' & \frac{c}{\sqrt{3}} & b\gamma \\ -\frac{c\kappa}{\sqrt{3}} - \sqrt{\frac{2}{3}}b\gamma\eta c' & \sqrt{\frac{2}{3}}\kappa c' - \frac{bc\gamma\eta}{\sqrt{3}} & \eta \\ \frac{c\eta}{\sqrt{3}} - \sqrt{\frac{2}{3}}b\gamma\kappa c' & -\frac{bc\gamma\kappa}{\sqrt{3}} - \sqrt{\frac{2}{3}}\eta c' & \kappa \end{bmatrix} \quad (\text{PMNS-II}) \quad (10)$$

where, $\gamma(\eta) = \eta^8$, $\kappa(\eta) = \cos \sin^{-1}(\eta)$ and $c' = (3 - c^2)^{\frac{1}{2}}/2$. Let us summarize some important features of the above two non familiar parametrization of PMNS matrix.

- This is to be highlighted that in either of the two possibilities **PMNS-I** or **PMNS II**, a vanishing reactor angle is not possible. If this is so, the mass eigenvalues will also disappear. Hence, the present parametrization cannot hold the Tri-Bimaximal (TB) and Bi-maximal (BM) framework in exact form, which assumes predominantly the reactor angle to be zero.
- The **PMNS-II** allows only the possibilities, $\theta_{23} > 45^\circ$ and $\theta_{12} \lesssim \sin^{-1}(1/\sqrt{3})$.
- The free parameter, $\epsilon \sim \mathcal{O}(\lambda)$ and $\eta \sim 4\lambda$, where λ is the Wolfenstein parameter.

Next, we try to understand how the ansatz in Eq. (2) will help to understand the texture of neutrino mass matrix? The ansatz reduces the number of free parameters, and the number of working parameters are less than that of physical ones. Hence we expect that the mass matrix is little predictive. To construct the same, we concentrate on the finding out some *exact* sum rules to relate different matrix elements.

Here, we construct the neutrino mass matrix, $\mathcal{M}_\nu = \mathcal{U} \mathcal{M}_\nu^d \mathcal{U}^T$. For a lucid flow of the present discussion we keep aside the numerical description of the internal parameters like ϵ , η etc. The details of the same can be found in the Table.(IV). We have the general texture of left-handed Majorana neutrino mass matrix, as shown below.

$$M_\nu \sim \begin{bmatrix} \mathcal{A} & \mathcal{B} & \mathcal{C} \\ \mathcal{B} & \mathcal{D} & \mathcal{E} \\ \mathcal{C} & \mathcal{E} & \mathcal{F} \end{bmatrix}. \quad (11)$$

If, the parametrization is ϵ -based, we have $\mathcal{M}_\nu = \mathcal{M}_\nu(\epsilon, d, f, s)$ and $\mathcal{M}_\nu = \mathcal{M}_\nu(\eta, b, c)$, if it is η -based. We start with the **NH-ND** case, parametrization of which is ϵ based. The matrix elements are tabulated in Table.(I). Choosing the working parameters, (ϵ, d, f, s) properly, we derive the following exact relations binding the matrix elements.

$$\text{Sum rule 1: } 2(\mathcal{A} + \mathcal{C}) - (\mathcal{E} - \mathcal{B}) = 0, \quad (12)$$

$$\text{Sum rule 2: } \mathcal{D} - (\mathcal{A} + \mathcal{F}) = 0, \quad (13)$$

$$\text{Sum rule 3: } 2\mathcal{A} - (\mathcal{B} + \mathcal{C}) = 0, \quad (14)$$

$$\text{Sum rule 4: } 2\mathcal{A} - (\mathcal{D} - \mathcal{E}) = 0, \quad (15)$$

These sum-rules promote a framework with,

$$\theta_{13} \approx 8.73^\circ, \quad \theta_{23} \approx 48.96^\circ, \quad \theta_{12} \approx 32.51^\circ. \quad (16)$$

Except the solar angle which is consistent with 2σ range (by sacrificing one of the sum rules, the solar angle can be made precise.), the rest lies within 1σ range. The above sum rules lead to the following texture,

$$M_\nu = \begin{bmatrix} \mathcal{A} & 2\mathcal{A} - \mathcal{C} & \mathcal{C} \\ 2\mathcal{A} - \mathcal{C} & 6\mathcal{A} + \mathcal{C} & 4\mathcal{A} + \mathcal{C} \\ \mathcal{C} & 4\mathcal{A} + \mathcal{C} & 5\mathcal{A} + \mathcal{C} \end{bmatrix} m_3, \quad (\text{Texture-I}) \quad (17)$$

where, $\mathcal{A} > \mathcal{C}$. It is interesting to note that the **Texture-I** is nothing but combination of a pattern akin to the modified Fritzsch-like texture of quark mass matrices [29, 30],

$$\begin{bmatrix} 0 & 2\mathcal{A} & 0 \\ 2\mathcal{A} & 6\mathcal{A} & 4\mathcal{A} \\ 0 & 4\mathcal{A} & 5\mathcal{A} \end{bmatrix}, \quad (18)$$

and, a μ - τ symmetric texture as shown below,

$$\begin{bmatrix} \mathcal{A} & -\mathcal{C} & \mathcal{C} \\ -\mathcal{C} & \mathcal{C} & \mathcal{C} \\ \mathcal{C} & \mathcal{C} & \mathcal{C} \end{bmatrix}. \quad (19)$$

Next, we turn towards another possibility, i.e., the **NHQD** scenario which is motivated by η -based parametrization. The matrix elements of the concerned neutrino mass matrix are illustrated in Table.(II).

As before, we encounter the following exact sum rules,

$$\text{Sum rule 1: } \mathcal{D} - \mathcal{F} - \mathcal{B} = 0, \quad (20)$$

$$\text{Sum rule 2: } 4\mathcal{E} - \mathcal{D} = 0, \quad (21)$$

$$\text{Sum rule 3: } 3\mathcal{C} - 2(\mathcal{D} - \mathcal{F}) = 0, \quad (22)$$

$$\begin{aligned}
\mathcal{A} &\approx -f^2 s\epsilon^4 + f^2 s\epsilon^3 - s\epsilon^4 + s\epsilon^2 + \epsilon^2 \\
\mathcal{B} &\approx -\frac{1}{2}d^2 f s\epsilon^4 - d s\epsilon^4 - \frac{d\epsilon^4}{2} + d\epsilon^2 - \frac{1}{2}f^3 s\epsilon^4 - \frac{1}{2}f s\epsilon^4 - f s\epsilon^3 + f s\epsilon^2 \\
\mathcal{C} &\approx -\frac{d^2\epsilon^3}{2} + d f s\epsilon^4 - d f s\epsilon^3 - f^2 s\epsilon^4 - s\epsilon^3 - \frac{\epsilon^3}{2} + \epsilon \\
\mathcal{D} &\approx -d^2 s\epsilon^3 - d^2\epsilon^4 + d^2\epsilon^2 - 2d f s\epsilon^4 + f^2 s\epsilon^4 - f^2 s\epsilon^3 + s\epsilon \\
\mathcal{E} &\approx \frac{1}{2}d^3 s\epsilon^4 - \frac{d^3\epsilon^3}{2} + d f^2 s\epsilon^4 - d s\epsilon^2 - d\epsilon^3 + d\epsilon + f s\epsilon^4 - f s\epsilon^3 \\
\mathcal{F} &\approx d^2 s\epsilon^3 + d^2\epsilon^4 - d^2\epsilon^2 + 2d f s\epsilon^4 + s\epsilon^4 + \frac{\epsilon^4}{4} - \epsilon^2 + 1
\end{aligned}$$

TABLE I. The elements of the general neutrino mass matrix (Normal Hierarchy-non-degenerate (**NH-ND**) case)

$$\begin{aligned}
\mathcal{A} &\approx b^2\gamma^2 + \frac{c^2\eta^2}{3} + \frac{2}{3}c\eta^2 c'^2 \\
\mathcal{B} &\approx -\frac{1}{3}\eta\{b\gamma(c^2\eta^2 - 3) + 2bc\gamma\eta^2 c'^2 + \sqrt{2}(c-1)c\eta\kappa c'\} \\
\mathcal{C} &\approx \frac{1}{3}\{b\gamma\kappa(3 - c^2\eta^2) - 2bc\gamma\eta^2\kappa c'^2 + \sqrt{2}(c-1)c\eta^3 c'\} \\
\mathcal{D} &\approx \frac{1}{9}\eta^2\{c(\sqrt{6}b\gamma\eta c' + \sqrt{3}\kappa c')^2 + 3(bc\gamma\eta - \sqrt{2}\kappa c')^2 + 9\} \\
\mathcal{E} &\approx -\frac{1}{3}\eta\{-2\eta^2\kappa(c')^2(b^2c\gamma^2 - 1) + \kappa(-b^2\gamma^2 c^2\eta^2 + c^3\eta^2 - 3) + \sqrt{2}b(c-1)c\gamma\eta c'(\eta^2 - \kappa^2)\} \\
\mathcal{F} &\approx \frac{1}{9}\eta^2(\sqrt{3}bc\gamma\kappa + \sqrt{6}\eta c')^2 + \frac{1}{9}c\eta^2(\sqrt{3}c\eta - \sqrt{6}b\gamma\kappa c')^2 + \kappa^2
\end{aligned}$$

TABLE II. The elements of the general neutrino mass matrix (**NH-QD** case)

which prescribe the following texture of the neutrino mass matrix guided by three parameters,

$$M_\nu = \begin{bmatrix} \mathcal{A} & \mathcal{B} & \frac{2}{3}\mathcal{B} \\ \mathcal{B} & 4\mathcal{E} & \mathcal{E} \\ \frac{2}{3}\mathcal{B} & \mathcal{E} & 4\mathcal{E} - \mathcal{B} \end{bmatrix} m_3, \quad (\text{Texture-II}) \quad (23)$$

with, $4\mathcal{E} > \mathcal{A} > \mathcal{E} > \mathcal{B}$. The above neutrino mass matrix is consistent with the prediction,

$$\theta_{12} \approx 34.08^\circ, \quad \theta_{23} \approx 49.66^\circ, \quad \theta_{13} \approx 10^\circ. \quad (24)$$

We see that the above framework predicts a reactor angle, lying slightly above the experimental observation. Here also, we experience a μ - τ symmetric texture with a perturbation matrix,

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} & \mathcal{B} \\ \mathcal{B} & 4\mathcal{E} & \mathcal{E} \\ \mathcal{B} & \mathcal{E} & 4\mathcal{E} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{1}{3}\mathcal{B} \\ 0 & 0 & 0 \\ \frac{1}{3}\mathcal{B} & 0 & -\mathcal{B} \end{bmatrix}. \quad (25)$$

A precise value $\theta_{13} \approx 9.3^\circ$, consistent within 1σ bound is obtainable at the cost of sacrificing the Sum rule 2 in Eq. (21) (See Fig. (2)). The other angles remain untouched. And, the corresponding texture of the neutrino mass matrix appears as in the following,

$$M_\nu = \begin{bmatrix} \mathcal{A} & \mathcal{B} & \frac{2}{3}\mathcal{B} \\ \mathcal{B} & \mathcal{D} & \mathcal{E} \\ \frac{2}{3}\mathcal{B} & \mathcal{E} & \mathcal{D} - \mathcal{B} \end{bmatrix} m_3, \quad (\text{Texture-III}) \quad (26)$$

with $\mathcal{D} > \mathcal{A} > \mathcal{E} > \mathcal{B}$. The discussion related to the hidden μ - τ symmetric texture is similar to that for **Texture-II**.

Similarly for the case of Inverted hierarchy (**IH-QD**), the matrix elements are shown in Table. (III). The realization of the following sum rules:

$$\text{Sum rule 1: } \mathcal{B} - \mathcal{C} = 0, \quad (27)$$

$$\text{Sum rule 2: } \mathcal{A} + \mathcal{E} - \mathcal{D} + \mathcal{B} = 0, \quad (28)$$

promote the neutrino mass matrix, M_ν , to assume the following texture,

$$M_\nu = \begin{bmatrix} \mathcal{A} & \mathcal{B} & \mathcal{B} \\ \mathcal{B} & \mathcal{D} & \mathcal{D} - \mathcal{A} - \mathcal{B} \\ \mathcal{B} & \mathcal{D} - \mathcal{A} - \mathcal{B} & \mathcal{F} \end{bmatrix} m_2, \quad (\text{Texture-IV}) \quad (29)$$

with $\mathcal{A} > \mathcal{F} > \mathcal{D} > |\mathcal{B}|$.

The above sum rules, restricts the reactor angle at,

$$\theta_{13} \approx 7.1^\circ, \quad (30)$$

Which is little lower than what we observe experimentally. The other predictions are,

$$\theta_{12} \approx 34.84^\circ, \quad \theta_{23} \approx 50.76^\circ. \quad (31)$$

which are consistent within 1σ . By changing the Sum rules a little, (See Fig. (2)) as in the following,

$$\text{Sum rule 1: } \mathcal{B} - \mathcal{C} = 0, \quad (32)$$

$$\text{Sum rule 2: } \mathcal{A} - \mathcal{B} - \mathcal{F} + \mathcal{E} = 0. \quad (33)$$

We can set the reactor angle within the 1σ bound ($\theta_{13} \approx 8.5^\circ$). The corresponding neutrino mass matrix appears as in the following,

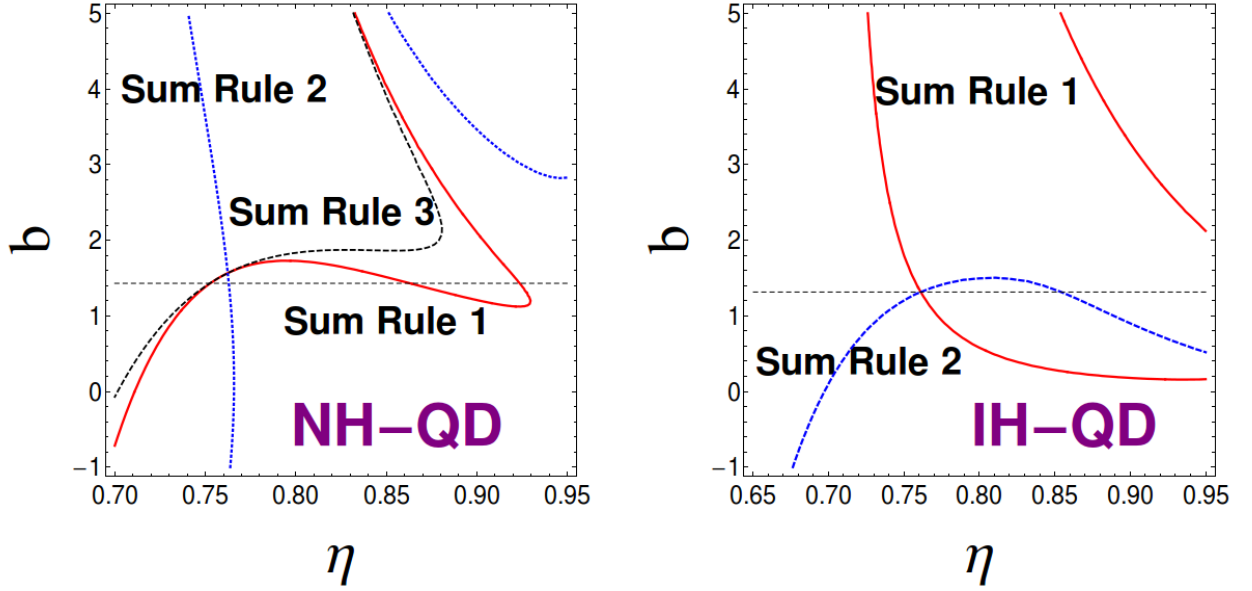


FIG. 2. The illustration of the Sum rules, in the plane b - η for the mass matrices corresponding to **Texture-II** (left) and **Texture-V** (right).

$\mathcal{A} \approx b^2\gamma^2\eta^2 + \frac{c^2}{3} + \frac{2}{3}cc'^2$
$\mathcal{B} \approx -\frac{1}{3}bc^2\gamma\eta - \frac{2}{3}bc\gamma\eta c'^2 + b\gamma\eta^3 + \frac{1}{3}\sqrt{2}c\kappa c' - \frac{1}{3}\sqrt{2}c^2\kappa c'$
$\mathcal{C} \approx -\frac{1}{3}b\gamma c^2\kappa - \frac{2}{3}b\gamma c\kappa c'^2 + b\gamma\eta^2\kappa - \frac{1}{3}\sqrt{2}c\eta c' + \frac{1}{3}\sqrt{2}c^2\eta c'$
$\mathcal{D} \approx \frac{1}{3}b^2c^2\gamma^2\eta^2 + \frac{2}{3}b^2c\gamma^2\eta^2c'^2 - \frac{2}{3}\sqrt{2}bc\gamma\eta\kappa c' + \frac{1}{3}\sqrt{2}2bc^2\gamma\eta\kappa c' + \frac{c^3\kappa^2}{3} + \frac{2}{3}\kappa^2c'^2 + \eta^4$
$\mathcal{E} \approx \frac{1}{3}b^2\gamma^2c^2\eta\kappa + \frac{2}{3}b^2\gamma^2c\eta\kappa c'^2 + \frac{1}{3}\sqrt{2}b\gamma c\eta^2c' - \frac{1}{3}\sqrt{2}b\gamma c\kappa^2c' - \frac{1}{3}\sqrt{2}b\gamma c^2\eta^2c' + \frac{1}{3}\sqrt{2}b\gamma c^2\kappa^2c' - \frac{1}{3}c^3\eta\kappa - \frac{2}{3}\eta\kappa c'^2 + \eta^3\kappa$
$\mathcal{F} \approx \frac{1}{3}b^2\gamma^2c^2\kappa^2 + \frac{2}{3}b^2\gamma^2c\kappa^2c'^2 + \frac{1}{3}\sqrt{2}2b\gamma c\eta\kappa c' - \frac{2}{3}\sqrt{2}b\gamma c^2\eta\kappa c' + \frac{c^3\eta^2}{3} + \frac{2}{3}\eta^2c'^2 + \eta^2\kappa^2$

TABLE III. The elements of the general neutrino mass matrix (**IH-QD** case)

$$M_\nu = \begin{bmatrix} \mathcal{A} & \mathcal{B} & \mathcal{B} \\ \mathcal{B} & \mathcal{D} & \mathcal{F} - \mathcal{A} + \mathcal{B} \\ \mathcal{B} & \mathcal{F} - \mathcal{A} + \mathcal{B} & \mathcal{F} \end{bmatrix} m_2, (\text{Texture-V}) \quad (34)$$

Where, $\mathcal{A} > \mathcal{F} > \mathcal{D} > |\mathcal{B}|$. We know that the TB mixing which is very often studied under several symmetry groups (e.g A_4 , S_4 and $\Delta(54)$ etc.), compels the elements of neutrino mass matrix to obey three sum rules [31–33] as shown below,

$$(M_\nu)_{12} = (M_\nu)_{13}, \quad (35)$$

$$(M_\nu)_{22} = (M_\nu)_{23}, \quad (36)$$

$$(M_\nu)_{11} + (M_\nu)_{12} = (M_\nu)_{22} + (M_\nu)_{23}. \quad (37)$$

One sees, that both **Texture IV** and **V** respects the first sum rule (See Eq. (35)), but breaks the second one (Eq. (36)). The sum rule in Eq. (37) is modified to,

$$(M_\nu)_{11} - (M_\nu)_{12} = (M_\nu)_{22} - (M_\nu)_{23}, \quad (38)$$

$$(M_\nu)_{12} - (M_\nu)_{11} = (M_\nu)_{23} - (M_\nu)_{33}, \quad (39)$$

for **Texture-IV** and **V** respectively.

In the above discussion, for all the five textures, the parameters are chosen in such a way that the two observational mass parameters can lie always within the 1σ boundary of experimental data. Also, it is seen that the number of independent matrix elements are 2, 3 or 4. Needless to say, that the textures displayed above are hierarchy dependent and exact. Also, the interesting fact that one sees is an unavoidable presence of a μ - τ symmetric or partially broken μ - τ symmetric textures in the above patterns. But this does not allow us to take the initial choice of M_ν as μ - τ symmetric, because both ϵ -based or η -based parametrization are reluctant to assume a vanishing θ_{13} (which is one of the important traces of the μ - τ symmetry). Certainly, some other efficient possibilities are to be contrived.

Although the five possible forms of neutrino mass matrix materialized above are model-independent, yet this is a natural quest whether the sum rules or the associated textures are motivated in flavor symmetries or not. To unveil the first principle working behind these sum rules, is beyond the scope of present note. Alternatively,

we try to put forward a noble way to feel the essence of possible symmetries by expressing individual mass matrices explicitly in terms of certain building block textures (see Table.(V)). Instead of describing how these building blocks appear we shall describe the physical situations where they may appear (this may help the model-builders to find out certain linkage). We see,

- The **Texture-I** is a linear superposition of H , f_1 , d_1 , e_4 and T_1 . The matrix H resembles a “Fritzsch-like” texture[29, 30, 34] which in general appears in the context of quarks. But, the same Fritzsch-like texture is obtainable for neutrinos also using S_3 symmetry[35]. The other matrices, f_1 , d_1 , e_4 which span the μ - τ symmetric part of **Texture-I**, are the members of S_4 symmetry group and belong to the same conjugacy class C_6 [36]. A partial realization of this μ - τ symmetric texture is experienced in some works motivated by S_4 [37] and A_4 [38] symmetry groups.
- The matrix d_1 again appears in **Textures II** and **III** and is followed by the identity matrix, I and the democratic matrix, D (derivable from S_3 symmetry). A linear combination of d_1 , D and I of such kind, in general contributes towards the mass matrix following TBM mixing and is also realized in different discrete symmetry based models. A framework with $A_4 \times Z_3$ symmetry[14, 15] is one such example.
- The matrix D and I also appear in **Texture-V**, supplemented by a matrix d_2 , which belongs to S_4 symmetry group[36]. On the other hand, in **Texture-IV**, D and I are replaced by c_3 and b_3 , which are the members of A_4 symmetry[36].
- The matrix T_1 , appears in **Texture-I, II, III** and

its presence is experienced in the neutrino mass matrix, where a deviation from TBM scenario [33] is taken into consideration. The matrix T_2 , plays an important role in the μ - τ symmetry violation for **Texture-II** and **III**. In Refs.[23, 39], one sees that to break the μ - τ symmetry of a neutrino mass matrix (concerning texture zeros), a matrix of following kind,

$$\begin{bmatrix} -k_2^2 & 0 & -k_2 \\ 0 & 0 & 0 \\ -k_2 & 0 & -1 \end{bmatrix}, \quad (40)$$

emerges and it may resemble T_2 , if we assume $k_2 = 1/3$.

- The matrices T_3 and T_4 , present in **Texture-IV** and **V** respectively, do not obey μ - τ symmetry. The parameters \mathcal{F} and \mathcal{D} are comparable and may obey the same in the limit, $\mathcal{F}/\mathcal{D} = 1$. In our case, this ratio is approximately 1.08. This little deviation from unity is needed to keep θ_{13} nonzero and to maintain the non-maximality of θ_{23} . A similar symmetry breaking operation of present kind with twisted parameters is motivated in certain extra dimension inspired theories [40].

In order to keep the discussion simple, all the parameters are treated as real and we hope that a further motivation in the same line is obtainable from the works in the Refs. [41–44].

To summarize, we have put forward an ansatz to relate the neutrino mixing angles and the mass ratios in the similar way it happens for quarks and discussed the phenomenological consequences. Finally we have associated the ansatz with certain sum rules to obtain possible patterns of neutrino mass matrix with exact texture, in the basis the charged lepton mass matrix is diagonal.

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	Parameters	Prediction	Texture
NH-ND	$\epsilon = 0.151859,$ $f = 3.53882,$ $d = 4.96783,$ $s = 1.21652,$ $m_3 = 0.0483055 \text{ eV}.$	$m_1 = 0.00135518 \text{ eV},$ $m_2 = 0.00892393 \text{ eV},$ $\Delta m_{21}^2 = 7.78 \times 10^{-5} \text{ eV}^2,$ $\Delta m_{31}^2 = 2.33 \times 10^{-3} \text{ eV}^2,$ $\sin^2 \theta_{12} = 0.289,$ $\sin^2 \theta_{23} = 0.569,$ $\sin^2 \theta_{13} = 0.0233,$ $\Sigma m_i = 0.0585846 \text{ eV}.$	$\begin{bmatrix} \mathcal{A} & 2\mathcal{A}-\mathcal{C} & \mathcal{C} \\ 2\mathcal{A}-\mathcal{C} & 6\mathcal{A}+\mathcal{C} & 4\mathcal{A}+\mathcal{C} \\ \mathcal{C} & 4\mathcal{A}+\mathcal{C} & 5\mathcal{A}+\mathcal{C} \end{bmatrix} m_3$ $\mathcal{A} = 0.0946855, \mathcal{C} = 0.038355.$
	$\eta = 0.762386,$ $b = 1.56568,$ $c = 0.96663,$ $m_3 = 0.0588 \text{ eV}.$	$m_1 = 0.0330417 \text{ eV},$ $m_2 = 0.0341791 \text{ eV},$ $\Delta m_{21}^2 = 7.65 \times 10^{-5} \text{ eV}^2,$ $\Delta m_{31}^2 = 2.37 \times 10^{-3} \text{ eV}^2,$ $\sin^2 \theta_{12} = 0.315,$ $\sin^2 \theta_{23} = 0.581,$ $\sin^2 \theta_{13} = 0.0319,$ $\Sigma m_i = 0.126036 \text{ eV}.$	$\begin{bmatrix} \mathcal{A} & \mathcal{B} & \frac{2}{3}\mathcal{B} \\ \mathcal{B} & 4\mathcal{E} & \mathcal{E} \\ \frac{2}{3}\mathcal{B} & \mathcal{E} & 4\mathcal{E}-\mathcal{B} \end{bmatrix} m_3$ $\mathcal{A} = 0.581819, \mathcal{B} = 0.0636482,$ $\mathcal{E} = 0.203162.$
NH-QD	$\eta = 0.756875,$ $b = 1.4945,$ $c = 0.96663,$ $m_3 = 0.0588 \text{ eV}.$	$m_1 = 0.0325638 \text{ eV},$ $m_2 = 0.0336859 \text{ eV},$ $\Delta m_{21}^2 = 7.43 \times 10^{-5} \text{ eV}^2,$ $\Delta m_{31}^2 = 2.40 \times 10^{-3} \text{ eV}^2,$ $\sin^2 \theta_{12} = 0.314,$ $\sin^2 \theta_{23} = 0.573,$ $\sin^2 \theta_{13} = 0.0259,$ $\Sigma m_i = 0.12506 \text{ eV}.$	$\begin{bmatrix} \mathcal{A} & \mathcal{B} & \frac{2}{3}\mathcal{B} \\ \mathcal{B} & \mathcal{D} & \mathcal{E} \\ \frac{2}{3}\mathcal{B} & \mathcal{E} & \mathcal{D}-\mathcal{B} \end{bmatrix} m_3$ $\mathcal{A} = 0.571196, \mathcal{B} = 0.058653,$ $\mathcal{D} = 0.80716, \mathcal{E} = 0.208885.$
IH-QD	$\eta = 0.774459,$ $b = 0.950921,$ $c = 0.989598,$ $m_2 = 0.0612 \text{ eV}.$	$m_1 = 0.0605614 \text{ eV},$ $m_3 = 0.0367071 \text{ eV},$ $\Delta m_{21}^2 = 7.75 \times 10^{-5} \text{ eV}^2,$ $\Delta m_{31}^2 = 2.32 \times 10^{-3} \text{ eV}^2,$ $\sin^2 \theta_{12} = 0.3264,$ $\sin^2 \theta_{23} = 0.599,$ $\sin^2 \theta_{13} = 0.0151,$ $\Sigma m_i = 0.158464 \text{ eV}.$	$\begin{bmatrix} \mathcal{A} & \mathcal{B} & \mathcal{B} \\ \mathcal{B} & \mathcal{D} & \mathcal{D}-\mathcal{A}-\mathcal{B} \\ \mathcal{B} & \mathcal{D}-\mathcal{A}-\mathcal{B} & \mathcal{F} \end{bmatrix} m_2$ $\mathcal{A} = 0.987095, \mathcal{B} = -0.0341297$ $\mathcal{D} = 0.761603, \mathcal{F} = 0.840778.$
	$\eta = 0.761187,$ $b = 1.31518,$ $c = 0.989553,$ $m_3 = 0.06047 \text{ eV}.$	$m_1 = 0.0598426 \text{ eV},$ $m_3 = 0.0350405 \text{ eV},$ $\Delta m_{21}^2 = 7.57 \times 10^{-5} \text{ eV}^2,$ $\Delta m_{31}^2 = 2.35 \times 10^{-3} \text{ eV}^2,$ $\sin^2 \theta_{12} = 0.3264,$ $\sin^2 \theta_{23} = 0.579,$ $\sin^2 \theta_{13} = 0.0219,$ $\Sigma m_i = 0.155355 \text{ eV}.$	$\begin{bmatrix} \mathcal{A} & \mathcal{B} & \mathcal{B} \\ \mathcal{B} & \mathcal{D} & \mathcal{F}-\mathcal{A}+\mathcal{B} \\ \mathcal{B} & \mathcal{F}-\mathcal{A}+\mathcal{B} & \mathcal{F} \end{bmatrix} m_2$ $\mathcal{A} = 0.983997, \mathcal{B} = -0.0430051$ $\mathcal{D} = 0.759459, \mathcal{F} = 0.825692,$

TABLE IV. The summary of parametrization and the neutrino mass matrix texture

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I	$\mathcal{A} \underbrace{\begin{bmatrix} 0 & 2 & 0 \\ 2 & 6 & 4 \\ 0 & 4 & 5 \end{bmatrix}}_H + \mathcal{C} \left\{ \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{f_1} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{d_1} + \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{e_4} \right\} + (\mathcal{A} - \mathcal{C}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{T_1}$
II	$(\mathcal{E} - \mathcal{B}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{d_1} + (4\mathcal{E} - \mathcal{B}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_I + \mathcal{B} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_D + (\mathcal{A} - 5\mathcal{E} + \frac{10}{9}\mathcal{B}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{T_1} + \mathcal{B} \underbrace{\begin{bmatrix} -\frac{1}{9} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 \\ -\frac{1}{3} & 0 & -1 \end{bmatrix}}_{T_2}$
III	$(\mathcal{E} - \mathcal{B}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{d_1} + (\mathcal{D} - \mathcal{B}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_I + \mathcal{B} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_D + (\mathcal{A} - \mathcal{E} - \mathcal{D} + \frac{10}{9}\mathcal{B}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{T_1} + \mathcal{B} \underbrace{\begin{bmatrix} -\frac{1}{9} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 \\ -\frac{1}{3} & 0 & -1 \end{bmatrix}}_{T_2}$
IV	$(\mathcal{A} - \mathcal{D}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}}_{d_2} - \mathcal{B} \left\{ \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}}_{c_3} + \underbrace{\begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{b_3} \right\} + \mathcal{D} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\mathcal{F}}{\mathcal{D}} \end{bmatrix}}_{T_3}$
V	$(\mathcal{A} - \mathcal{F}) \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}}_{d_2} + \mathcal{B} \left\{ \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_D - \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_I \right\} + \mathcal{F} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\mathcal{D}}{\mathcal{F}} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_4}$

TABLE V. The expression of **Textures I-V**, in terms of corresponding building block matrices

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